A3

ddl: 2023/10/8 23:59

Note: $\lg n$ means $\log_2 n$ in this assignment.

1 Solving Recurrence

1.1 The Substitution Method

(From CLRS 4.3-1)

Use the substitution method to show that each of the following recurrences defined on the reals has the asymptotic solution specified:

1. $T(n) = 2T(n/2 + 17) + n$ has solution $T(n) = O(n \lg n)$.

2. $T(n) = 2T(n/3) + \Theta(n)$ has solution $T(n) = \Theta(n)$.

Note: when proving the solution, it is helpful to read the *Avoiding pitfalls* section of CLRS 4.3.

1.2 The Recursion-tree Method

1.2.1 4.4-4 of CLRS

Use a recursion tree to justify a good guess for the solution to the recurrence $T(n) = T(\alpha n) + T((1 - \alpha)n) + \Theta(n)$, where α is a constant in the range $0 < \alpha < 1$.

1.2.2 1.8 of *Algorithms* by Erickson

Use recursion trees to solve each of the following recurrences:

1. $N(n) = 2N(n/2) + O(n/\lg n)$

2. $Q(n) = \sqrt{2n}Q(\sqrt{2n}) + \sqrt{n}$

Note: A rigorous proof is not required, but an **illustration of your ideas** is neccessary.

1.3 The Master Theorem

(From CLRS 4.5-1)

Use the master method to give tight asymptotic bounds for the following recurrences:

1. $T(n) = 2T(n/4) + \sqrt{n} \lg^2 n$

2. $T(n) = 2T(n/4) + n^2$

2 Heap

2.1 Heap Sort Running Time

(From CLRS 6.4-5)

```
HEAPSORT(A, n)
BUILD-MAX-HEAP(A, n)
for i = n downto 2
    exchange A[1] with A[i]
    A.heap-size = A.heap-size − 1
    MAX-HEAPIFY(A, 1)
```

Given the psedocode of HEAPSORT, show that when all the elements of A are distinct, the best-case running time of HEAPSORT is $\Omega(n \lg n)$.