The height expectation of skip list

- For each level $i \in \{1, 2, 3, ..., \infty\}$, define the indicator random variable:
  - $I_i = \begin{cases} 0 & \text{if the current level } L_i \text{ is empty} \\ 1 & \text{if if the current level } L_i \text{ is non-empty} \end{cases}$ ➡ The height h of the skip-list is then given by $h = \sum_{i=1}^{\infty} I_i$
- Note: I_i is never more than the length (number of elements) of L_i, hence, $\mathbb{E}(I_i) \le \mathbb{E}(|L_i|) = \frac{n}{2^i}$
- At last, we have $\mathbb{E}(h) = \mathbb{E}\left[\sum_{i=1}^{\infty} I_i\right] = \sum_{i=0}^{\infty} \mathbb{E}[I_i] = \sum_{i=1}^{\lfloor \lg n \rfloor} \mathbb{E}[I_i] + \sum_{i=\lfloor \lg n \rfloor+1}^{\infty} \mathbb{E}[I_i]$

$$\le \sum_{i=1}^{\lfloor \lg n \rfloor} 1 + \sum_{i=\lfloor \lg n \rfloor+1}^{\infty} \frac{n}{2^i} = \lfloor \lg n \rfloor + \sum_{i=\lfloor \lg n \rfloor+1}^{\infty} \frac{1}{2^{i-\lg n}} \le \lg n + \sum_{i=0}^{\infty} \frac{1}{2^i} = \lg n + 2$$

I_i is indicator random variable